Esercizi svolti

Forme canoniche SP e PS

1. Data seguente tabella di verità determinare la forme canoniche SP e PS

C	B	A	F	$\overline{F}$	Mintermini	Maxtermini
0	0	0	0	1	$\overline{C}\,\overline{B}\,\overline{A}=m0$	$C+B+A=M0$
0	0	1	0	1	$\overline{C}\,\overline{B}\,A=m1$	$C+B+\overline{A}=M1$
0	1	0	0	1	$\overline{C}\,B\,\overline{A}=m2$	$C+\overline{B}+A=M2$
0	1	1	1	0	$\overline{C}\,B\,A=m3$	$C+\overline{B}+\overline{A}=M3$
1	0	0	1	0	$C\,\overline{B}\,\overline{A}=m4$	$\overline{C}+B+A=M4$
1	0	1	1	0	$C\,\overline{B}\,A=m5$	$\overline{C}+B+\overline{A}=M0$
1	1	0	1	0	$C\,B\,\overline{A}=m6$	$\overline{C}+\overline{B}+A=M6$
1	1	1	1	0	$C\,B\,A=m7$	$\overline{C}+\overline{B}+\overline{A}=M7$

Forme SP per $F\ e\overline{F}$

$$F(C,B,A)=AB\overline{C}+\overline{A}\,\overline{B}C+A\overline{B}C+\overline{A}\,BC+ABC$$
$$F(C,B,A)=m3+m4+m5+m6+m7$$
$$F(C,B,A)=\sum m(3,4,5,6,7)$$

$$\overline{F}(C,B,A)=\overline{A}\,\overline{B}\overline{C}+A\overline{B}\overline{C}+\overline{A}\,B\overline{C}$$
$$\overline{F}(C,B,A)=m0+m1+m2$$
$$\overline{F}(C,B,A)=\sum m(0,1,2)$$

Forme PS per $F\ e\overline{F}$

$$F(C,B,A)=(A+B+C)(\overline{A}+B+C)(A+\overline{B}+C)$$
$$F(C,B,A)=M0\ M1\ M2$$
$$F(C,B,A)=\prod M(0,1,2)$$

$$\overline{F}(C,B,A)=(\overline{A}+\overline{B}+C)(A+B+\overline{C})(\overline{A}+B+\overline{C})(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+\overline{C})$$
$$\overline{F}(C,B,A)=M3\ M4\ M5\ M6\ M7$$
$$\overline{F}(C,B,A)=\prod M(3,4,5,6,7)$$

Ricavare la forma PS di F dalla forma SP di $\overline{F}$

$$\overline{F}(C,B,A)=\overline{A}\,\overline{B}\overline{C}+A\overline{B}\overline{C}+\overline{A}\,B\overline{C}$$
$$F(C,B,A)=\overline{\overline{A}\,\overline{B}\overline{C}+A\overline{B}\overline{C}+\overline{A}\,B\overline{C}}$$
$$F(C,B,A)=\overline{\overline{A}\,\overline{B}\overline{C}}\ \ \overline{A\overline{B}\overline{C}}\ \ \overline{\overline{A}\,B\overline{C}}$$

$F(C,B,A)=(A+B+C)(\overline{A}+B+C)(A+\overline{B}+C)$

Ricavare la forma SP di F dalla forma PS di $\overline{F}$

$\overline{F}(C,B,A)=(C+\overline{B}+\overline{A})(\overline{C}+B+A)(\overline{C}+B+\overline{A})(\overline{C}+\overline{B}+A)(\overline{C}+\overline{B}+\overline{A})$

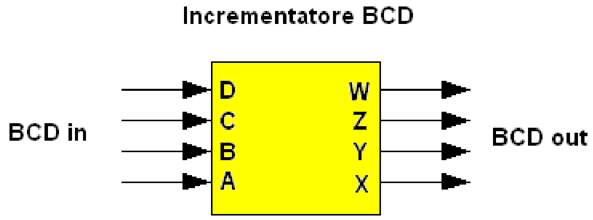
$\overline{\overline{F}}(C,B,A)=\overline{(C+\overline{B}+\overline{A})(\overline{C}+B+A)(\overline{C}+B+\overline{A})(\overline{C}+\overline{B}+A)(\overline{C}+\overline{B}+\overline{A})}$

$F(C,B,A)=\overline{(C+\overline{B}+\overline{A})}+\overline{(\overline{C}+B+A)}+\overline{(\overline{C}+B+\overline{A})}+\overline{(\overline{C}+\overline{B}+A)}+\overline{(\overline{C}+\overline{B}+\overline{A})}$

$F(C,B,A)=(\overline{C}\,B\,A)+(C\,\overline{B}\,\overline{A})+(C\,\overline{B}\,A)+(C\,B\,\overline{A})+(C\,B\,A)$

2. Realizzare la sintesi di un Incrementatore BCD

D	C	B	A	W	Z	Y	X
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	-	-	-	-
1	0	1	1	-	-	-	-
1	1	0	0	-	-	-	-
1	1	0	1	-	-	-	-
1	1	1	0	-	-	-	-
1	1	1	1	-	-	-	-



$X = m0 + m2 + m4 + m6 + m8 + d10 + d11 + d12 + d13 + d14 + d15$

$X = M1\ M3\ M5\ M7\ M9\ D10\ D11\ D12\ D13\ D14\ D15$

$Y = m1 + m2 + m5 + m6 + d10 + d11 + d12 + d13 + d14 + d15$

$Y = M0\ M3\ M4\ M7\ M8\ M9\ D10\ D11\ D12\ D13\ D14\ D15$

$Z = m3 + m4 + m5 + m6 + d10 + d11 + d12 + d13 + d14 + d15$

$Z = M0\ M1\ M2\ M7\ M8\ M9\ D10\ D11\ D12\ D13\ D14\ D15$

$W = m7 + m8 + d10 + d11 + d12 + d13 + d14 + d15$

$W = M0\ M1\ M2\ M3\ M4\ M5\ M6\ M9\ D10\ D11\ D12\ D13\ D14\ D15$

$W(D,C,B,A)=\sum m(7,8)+d(10,11,12,13,14,15)$

$Z(D,C,B,A)=\sum m(3,4,5,6)+d(10,11,12,13,14,15)$

$Y(D,C,B,A)=\sum m(1,2,5,6)+d(10,11,12,13,14,15)$

$X(D,C,B,A)=\sum m(0,2,4,6,8)+d(10,11,12,13,14,15)$

W

DC \ BA	00	01	11	10
00	[0] **0**	[1] **0**	[3] **0**	[2] **0**
01	[4] **0**	[5] **0**	[7] **1**	[6] **0**
11	[12] **-**	[13] **-**	[15] **-**	[14] **-**
10	[8] **1**	[9] **0**	[11] **-**	[10] **-**

ABC

$\overline{A}D$

$$W = \overline{A}D + ABC$$

Z

DC \ BA	00	01	11	10
00	[0] **0**	[1] **0**	[3] **1**	[2] **0**
01	[4] **1**	[5] **1**	[7] **0**	[6] **1**
11	[12] **-**	[13] **-**	[15] **-**	[14] **-**
10	[8] **0**	[9] **0**	[11] **-**	[10] **-**

$AB\overline{C}$

$\overline{A}C$

$\overline{B}C$

$$Z = \overline{A}C + \overline{B}C + AB\overline{C}$$

Y

DC \ BA	00	01	11	10
00	[0] **0**	[1] **1**	[3] **0**	[2] **1**
01	[4] **0**	[5] **1**	[7] **0**	[6] **1**
11	[12] **-**	[13] **-**	[15] **-**	[14] **-**
10	[8] **0**	[9] **0**	[11] **-**	[10] **-**

$A\overline{B}\overline{D}$

$\overline{A}B$

$$Y = A\overline{B}\overline{D} + \overline{A}B$$

X

DC \ BA	00	01	11	10
00	[0] **1**	[1] **0**	[3] **0**	[2] **1**
01	[4] **1**	[5] **0**	[7] **0**	[6] **1**
11	[12] **-**	[13] **-**	[15] **-**	[14] **-**
10	[8] **1**	[9] **0**	[11] **-**	[10] **-**

$\overline{A}$

$$X = \overline{A}$$

Mappe di Karnaugh

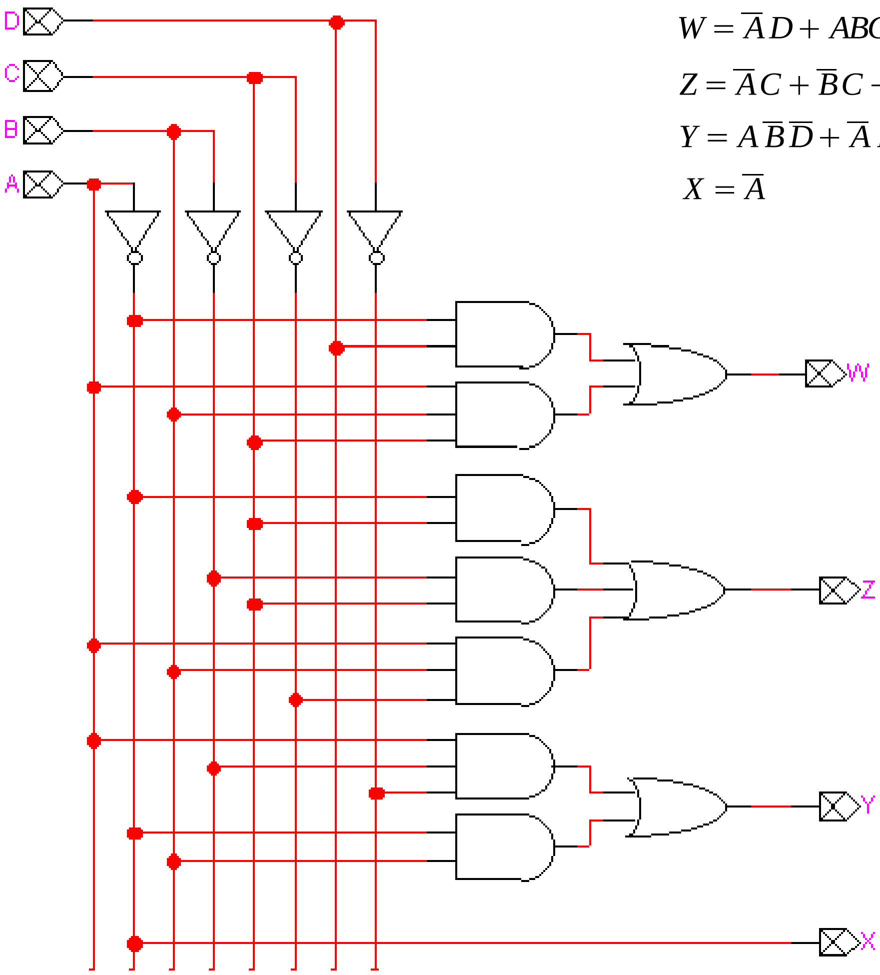
Ingresso BCD Schema logico Uscita BCD

$$W = \overline{A}D + ABC$$

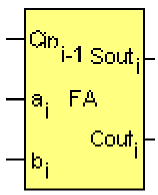
$$Z = \overline{A}C + \overline{B}C + AB\overline{C}$$

$$Y = A\overline{B}\,\overline{D} + \overline{A}B$$

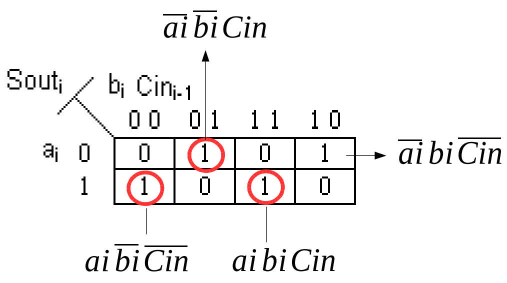
$$X = \overline{A}$$



3. Effettuare la sintesi minima di un FullAdder



Full Adder				
a_i	b_i	Cin_{i-1}	$Sout_i$	$Cout_i$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



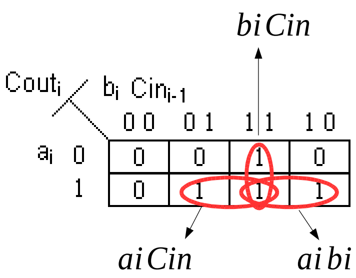
$$Sout = ai\,\overline{bi}\,\overline{Cin} + \overline{ai}\,\overline{bi}\,Cin + ai\,bi\,Cin + \overline{ai}\,bi\,\overline{Cin}$$

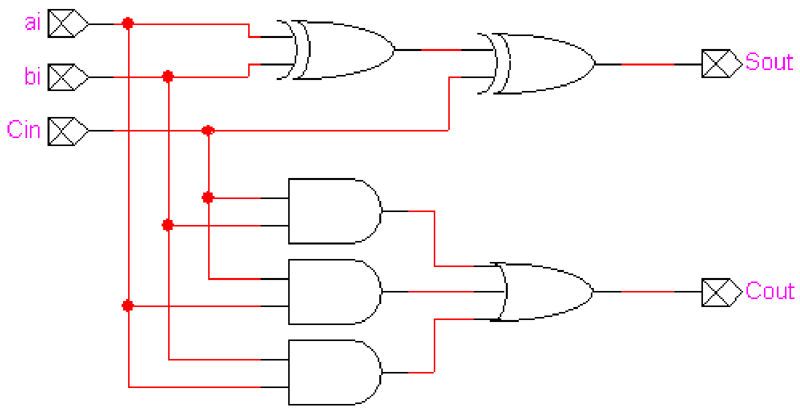
$$Sout = \overline{bi}\,(ai\,\overline{Cin} + \overline{ai}\,Cin) + bi\,(ai\,Cin + \overline{ai}\,\overline{Cin})$$

$$Sout = \overline{bi}\,(ai \oplus Cin) + bi\,\overline{(ai \oplus Cin)}$$

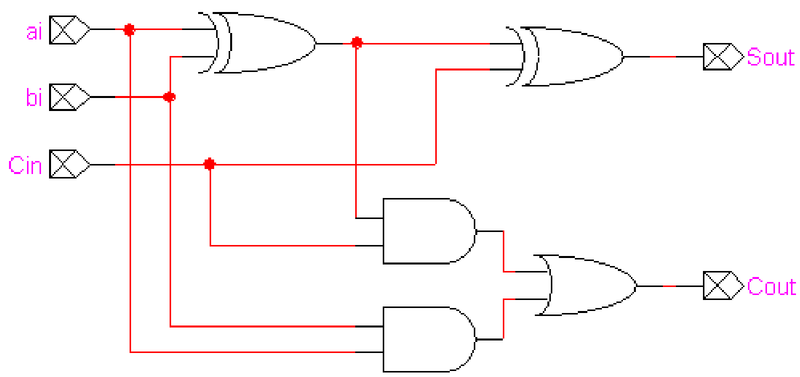
$$Sout = ai \oplus bi \oplus Cin$$

$$Cout = ai\,Cin + bi\,Cin + ai\,bi$$





La figura rappresenta lo schema logico relativo alle funzioni Sout e Cout.



Lo schema logico associato alla funzione Cout puo' in parte sfruttare la rete relativa alla funzione Sout. In tal modo rispetto alla soluzione precedente si fa uso di una porta AND in meno e si impiega una porta OR a due ingressi piuttosto che a tre.

L'espressione combinatoria associata all'uscita Cout della rete ottimizzata risulta essere logicamente equivalente all'espressione ricavata direttamente dalle K-mappe:

$$Cout = Cin(ai \oplus bi) + ai\,bi \quad \Rightarrow \quad Cout = Cin(ai\,\overline{bi} + \overline{ai}\,bi) + ai\,bi$$

Si aggiungono i termini

$$ai\,bi\,Cin + ai\,bi\,Cin$$

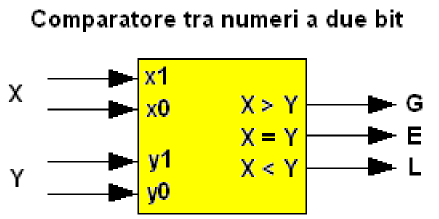
poiche' e' gia presente $ai\,bi$

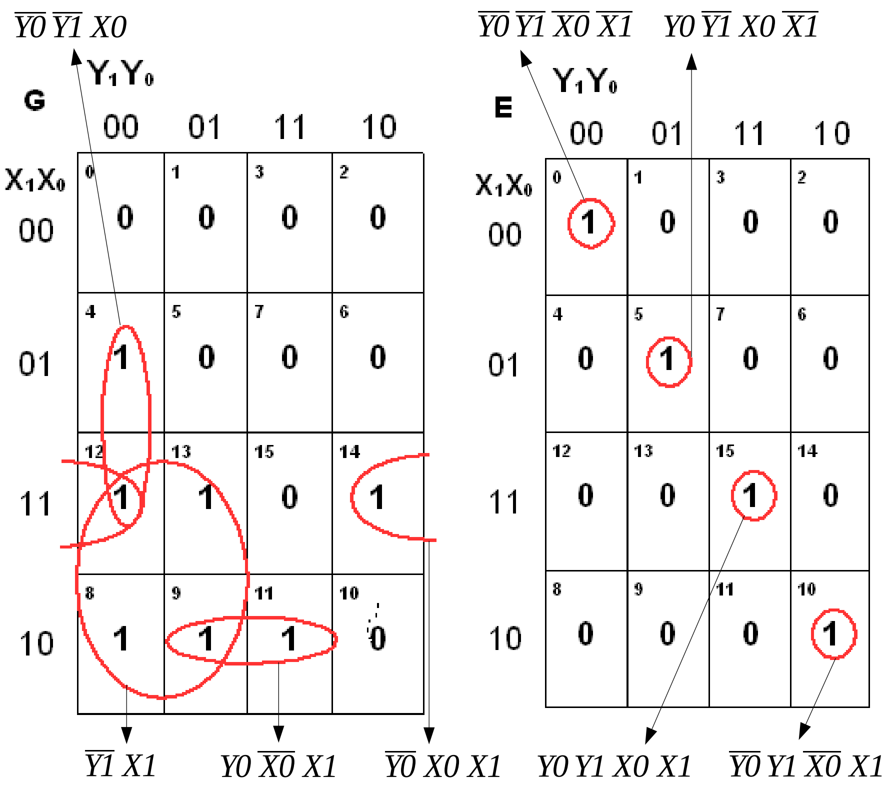
$$Cout = ai\,\overline{bi}\,Cin + \overline{ai}\,bi\,Cin + ai\,bi \quad \Rightarrow \quad Cout = ai\,\overline{bi}\,Cin + ai\,bi\,Cin + \overline{ai}\,bi\,Cin + ai\,bi\,Cin + ai\,bi$$

$$Cout = ai\,Cin + bi\,Cin + ai\,bi$$

4. Effettuare la sintesi minima di un comparatore tra numeri binari a due bit.

X_1	X_0	Y_1	Y_0	G	E	L
0	0	0	0	0	1	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0





L: X_1X_0 \ Y_1Y_0	00	01	11	10
00	^{0} 0	^{1} 1	^{3} 1	^{2} 1
01	^{4} 0	^{5} 0	^{7} 1	^{6} 1
11	^{12} 0	^{13} 0	^{15} 0	^{14} 0
10	^{8} 0	^{9} 0	^{11} 0	^{10} 0

$Y0\,\overline{X0}\,\overline{X1}$

$Y1\,\overline{X1}$

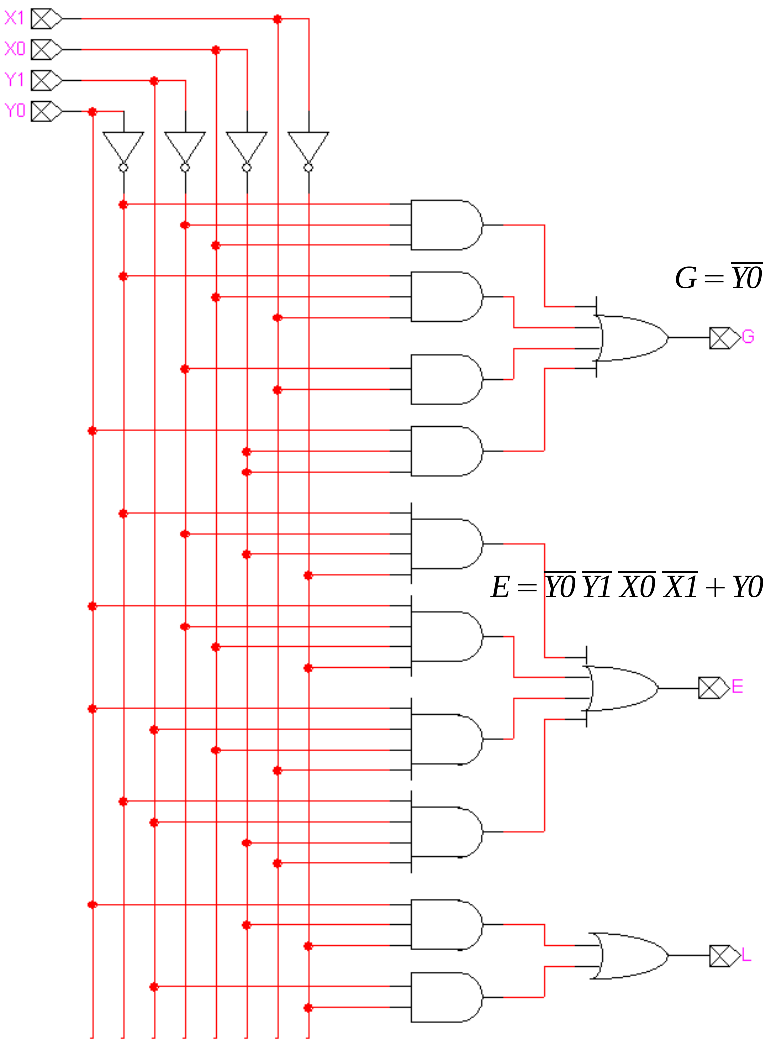
$$G = \overline{Y0}\,\overline{Y1}\,X0 + \overline{Y0}\,X0\,X1 + \overline{Y1}\,X1 + Y0\,\overline{X0}\,X1$$

$$E = \overline{Y0}\,\overline{Y1}\,\overline{X0}\,\overline{X1} + Y0\,\overline{Y1}\,X0\,\overline{X1} + Y0\,Y1\,X0\,X1 + \overline{Y0}\,Y1\,\overline{X0}\,X1$$

$$L = Y0\,\overline{X0}\,\overline{X1} + Y1\,\overline{X1}$$

Implementazione a due livelli del comparatore binario a due bit.

La rete logica impiega 13 porte :
4 porte AND a 4 ingressi;
4 porte AND a 3 ingressi;
2 porte AND a 2 ingressi;
2 porte OR a 4 ingressi;
1 porte OR a 2 ingressi;



$$G = \overline{Y0}\,\overline{Y1}\,X0 + \overline{Y0}\,X0\,X1 + \overline{Y1}\,X1 + Y0\,\overline{X0}\,X1$$

$$E = \overline{Y0}\,\overline{Y1}\,\overline{X0}\,\overline{X1} + Y0\,\overline{Y1}\,X0\,\overline{X1} + Y0\,Y1\,X0\,X1 + \overline{Y0}\,Y1\,\overline{X0}\,X1$$

$$L = Y0\,\overline{X0}\,\overline{X1} + Y1\,\overline{X1}$$

$$E = \overline{Y0}\,\overline{Y1}\,\overline{X0}\,\overline{X1} + Y0\,\overline{Y1}\,X0\,\overline{X1} + Y0\,Y1\,X0\,X1 + \overline{Y0}\,Y1\,\overline{X0}\,X1$$

$$E = \overline{Y1}\,\overline{X1}\,(\overline{Y0}\,\overline{X0} + Y0\,X0) + Y1X1\,(Y0\,X0 + \overline{Y0}\,\overline{X0})$$

$$E = (\overline{Y0}\,\overline{X0} + Y0\,X0)(\overline{Y1}\,\overline{X1} + Y1X1)$$

$$E = (Y0 \odot X0)(Y1 \odot X1)$$

$$G = \overline{Y0}\,\overline{Y1}\,X0 + \overline{Y0}\,X0\,X1 + \overline{Y1}\,X1 + Y0\,\overline{X0}\,X1$$

$$G = X1\,(X0 \oplus Y0) + \overline{Y1}\,(X1 + \overline{Y0}\,X0)$$

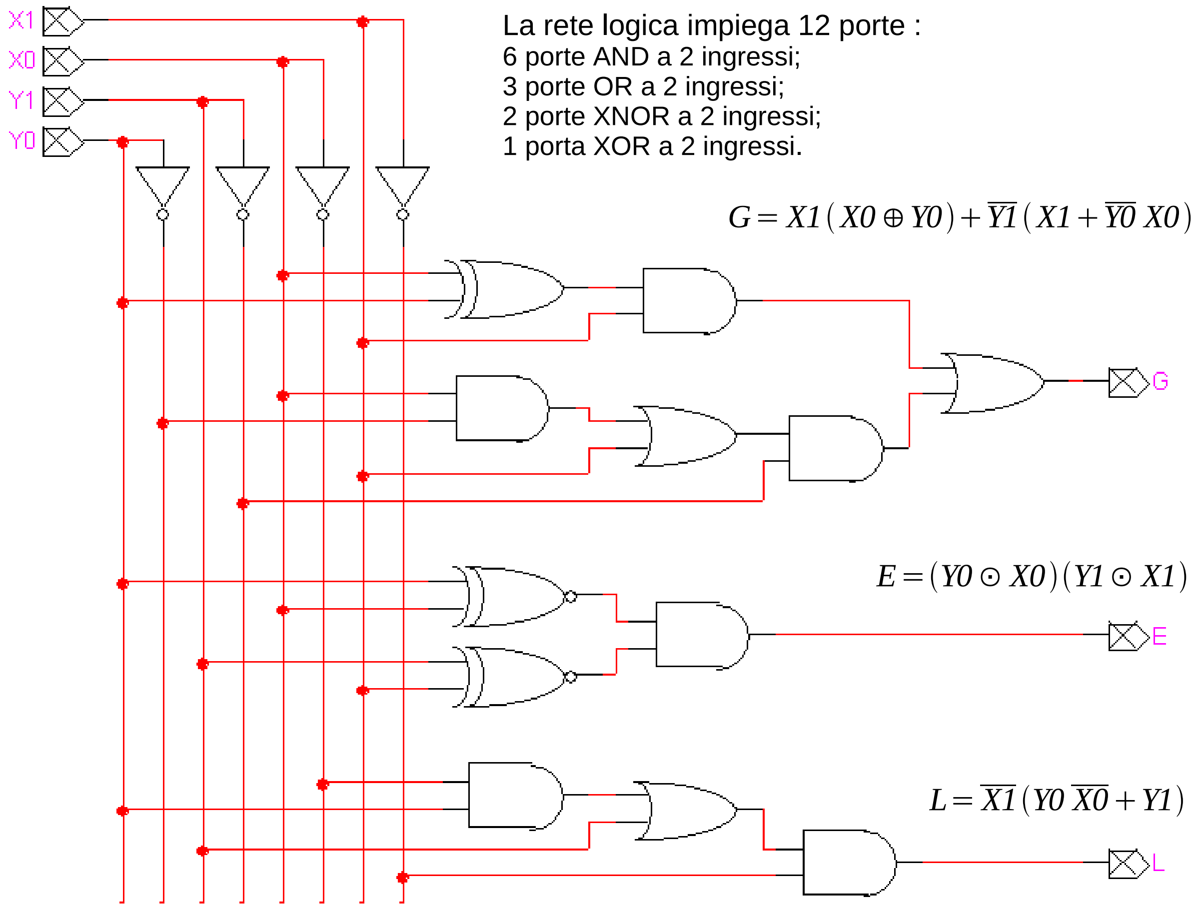
$$L = Y0\,\overline{X0}\,\overline{X1} + Y1\,\overline{X1}$$

$$L = \overline{X1}\,(Y0\,\overline{X0} + Y1)$$

Per effettuare una implementazione multilivello è possibile ricavare E, G ed L in forma fattorizzata. Cio' comporta una riduzione nel numero di ingressi delle porte impiegate ed un aumento del numero dei livelli della rete, quindi un aumento del tempo necessario ai segnali per attraversare la rete (tempo di propagazione).

Implementazione multilivello del comparatore tra numeri binari a due bit.

La rete logica impiega 12 porte :
6 porte AND a 2 ingressi;
3 porte OR a 2 ingressi;
2 porte XNOR a 2 ingressi;
1 porta XOR a 2 ingressi.



5. Semplificare le seguente funzione utilizzando le mappe di Karnaugh

$$F(W,V,Z,Y,X)=\prod M(0,4,18,19,22,23,25,29)$$

$(W+X+Y+V)$ $(\overline{W}+\overline{Y}+V)$

W = 0

F (VZ \ YX)	00	01	11	10
00	(0) 0	(1) 1	(3) 1	(2) 1
01	(4) 0	(5) 1	(7) 1	(6) 1
11	(12) 1	(13) 1	(15) 1	(14) 1
10	(8) 1	(9) 1	(11) 1	(10) 1

W = 1

F (VZ \ YX)	00	01	11	10
00	(16) 1	(17) 1	(19) 0	(18) 0
01	(20) 1	(21) 1	(23) 0	(22) 0
11	(28) 1	(29) 0	(31) 1	(30) 1
10	(24) 1	(25) 0	(27) 1	(26) 1

$(\overline{W}+\overline{X}+Y+\overline{V})$

$$F=(W+X+Y+V)(\overline{W}+\overline{Y}+V)(\overline{W}+\overline{X}+Y+\overline{V})$$

Forma minima normale PS

$X\,\overline{Y}\,\overline{V}$ $\overline{W}\,Y$ $W\,\overline{X}\,\overline{Y}$ YV $\overline{W}\,V$

W = 0

F (VZ \ YX)	00	01	11	10
00	(0) 0	(1) 1	(3) 1	(2) 1
01	(4) 0	(5) 1	(7) 1	(6) 1
11	(12) 1	(13) 1	(15) 1	(14) 1
10	(8) 1	(9) 1	(11) 1	(10) 1

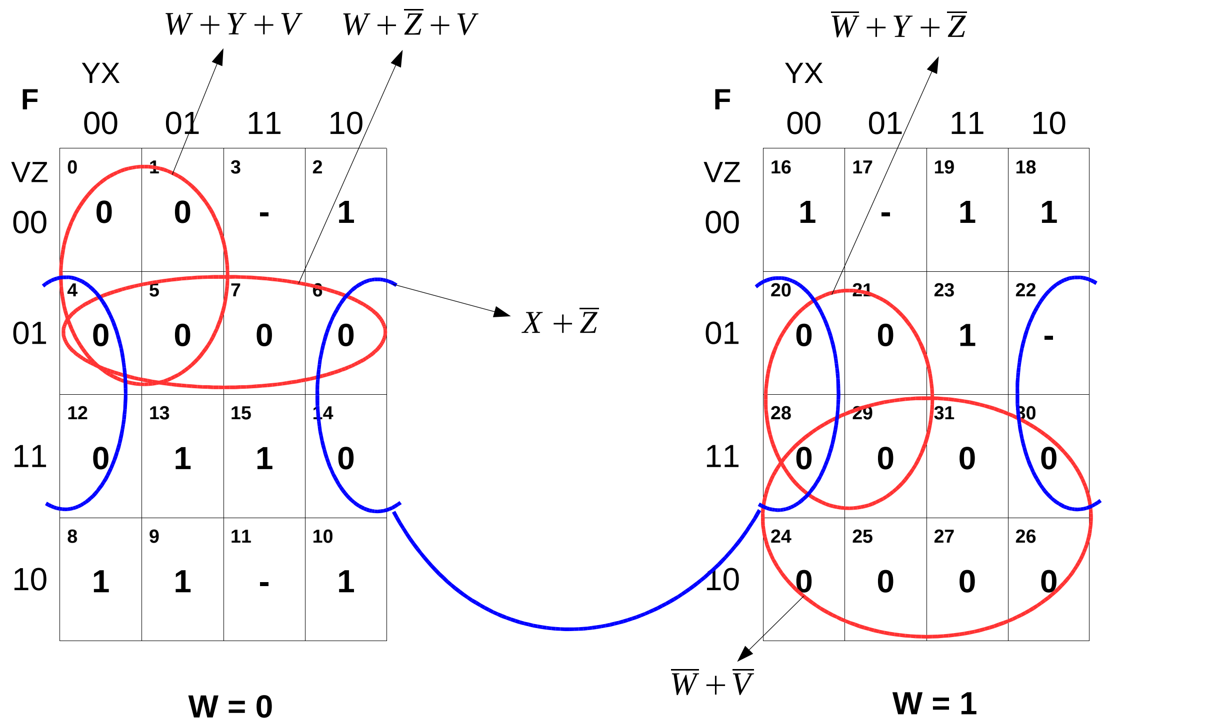
W = 1

F (VZ \ YX)	00	01	11	10
00	(16) 1	(17) 1	(19) 0	(18) 0
01	(20) 1	(21) 1	(23) 0	(22) 0
11	(28) 1	(29) 0	(31) 1	(30) 1
10	(24) 1	(25) 0	(27) 1	(26) 1

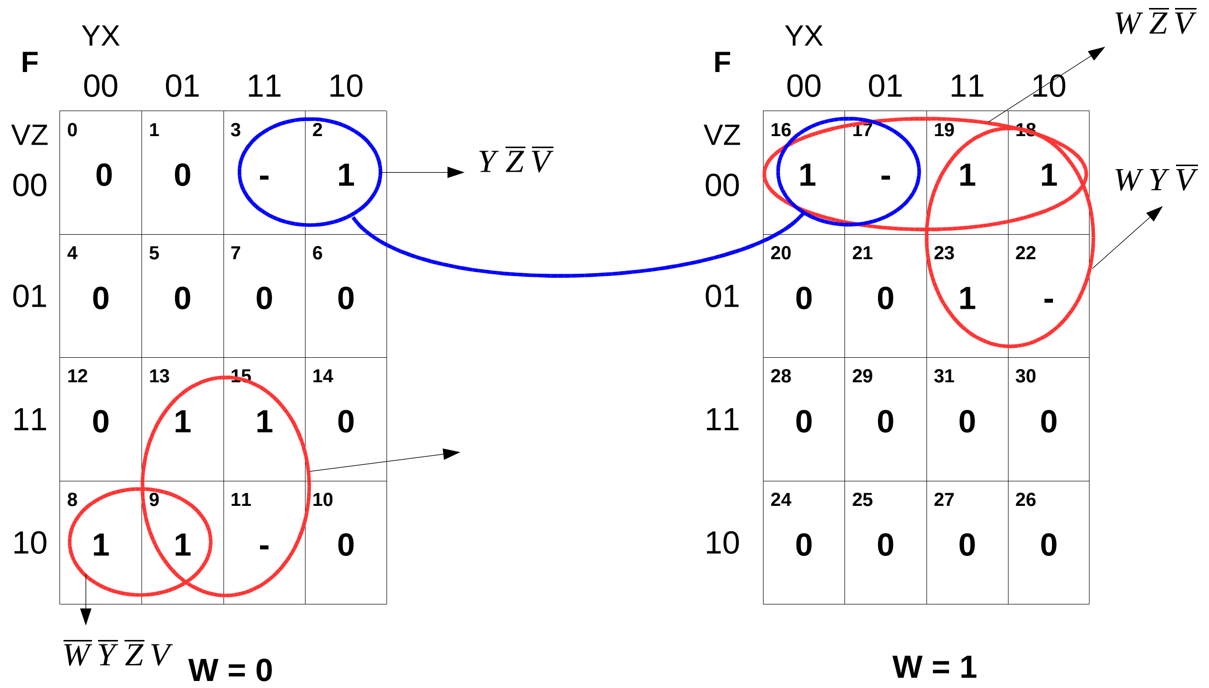
$$F=X\,\overline{Y}\,\overline{V}+\overline{W}\,V+\overline{W}\,Y+YV+W\,\overline{X}\,\overline{Y}$$

Forma minima normale SP

$$F(W,V,Z,Y,X)=\sum m(2,8,9,10,13,15,16,18,19,23)+\sum d(3,11,17,22)$$



$F=(W+Y+V)(W+\overline{Z}+V)(X+\overline{Z})(\overline{W}+Y+\overline{Z})(\overline{W}+\overline{V})$ — Forma minima normale PS



$F=\overline{W}\,\overline{Y}\,\overline{Z}\,V+\overline{W}\,X\,V+Y\,\overline{Z}\,\overline{V}+W\,\overline{Z}\,\overline{V}+W\,Y\,\overline{V}$ — Forma minima normale SP